1.

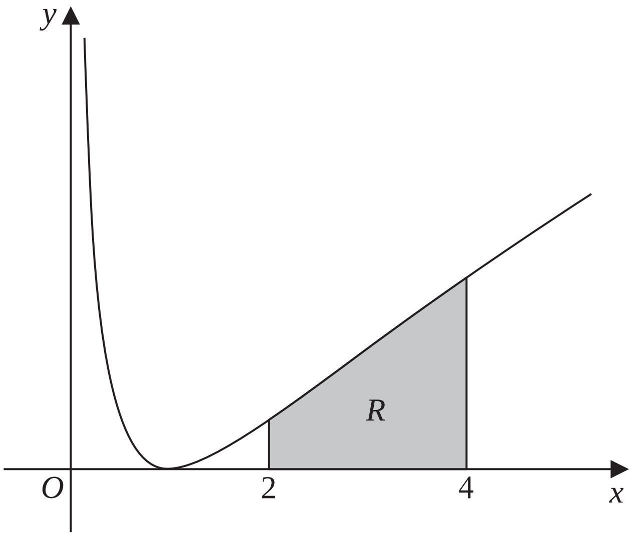

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \qquad x > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

The table below shows corresponding values of x and y, with the values of y given to 4 decimal places.

$4-3.5=0.5$ $h=0.5$ ①

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of R, giving your answer to 3 significant figures.

(3)

(b) Use algebraic integration to find the exact area of R, giving your answer in the form

$$y = a(\ln 2)^2 + b\ln 2 + c$$

where a, b and c are integers to be found.

(5)

a) $R = \int_2^4 (\ln x)^2 dx \approx \frac{0.5}{2}\left[0.4805 + 1.9218 + 2(0.8396 + 1.2069 + 1.5694)\right]$ ①

$= 2.41$ (3sf) ①

b) $R = \int_2^4 (\ln x)^2 \, dx$

by parts: $\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$

find $\int (\ln x)^2 \, dx$

let $u = (\ln x)^2$ $\quad \frac{dv}{dx} = 1$

$\frac{du}{dx} = \frac{2 \ln x}{x}$ $\quad v = x$ (1)

$\int (\ln x)^2 \, dx = x(\ln x)^2 - 2\int \ln x \, dx$ (1)

$\int \ln x \, dx$

let $u = \ln x$ $\quad \frac{dv}{dx} = 1$

$\frac{du}{dx} = \frac{1}{x}$ $\quad v = x$

$\int \ln x \, dx = x \ln x - \int 1 \, dx$

$= x \ln x - x$

$\int (\ln x)^2 \, dx = x(\ln x)^2 - 2(x \ln x - x)$

$= x(\ln x)^2 - 2x \ln x + 2x$ (1)

$R = \int_2^4 (\ln x)^2 \, dx = \left[x(\ln x)^2 - 2x \ln x + 2x \right]_2^4$

$= 4[\ln 4]^2 - 8\ln 4 + 8 - 2(\ln 2)^2 + 4\ln 2 - 4$

$= 4[2\ln 2]^2 - 16\ln 2 - 2[\ln 2]^2 + 4\ln 2 + 4$ (1)

$= 16(\ln 2)^2 - 12\ln 2 - 2(\ln 2)^2 + 4$

$= 14(\ln 2)^2 - 12\ln 2 + 4$ (1)

$\ln 4 = \ln 2^2 = 2\ln 2$

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Show that

$$\int_1^{e^2} x^3 \ln x \, dx = ae^8 + b$$

where a and b are rational constants to be found.

(5)

$u = \ln x \implies \frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}\,dx$

$dv = x^3 \implies v = \frac{1}{4}x^4$

$\int x^3\,dx = \frac{1}{4}x^4$

integration by parts:

$\int u\,dv = uv - \int v\,du$

choose $\ln x$ as u because it is much easier to differentiate than integrate.

$\int u\,dv = uv - \int v\,du$

$$\int_1^{e^2} x^3 \ln x\,dx = \left[\ln x \times \frac{1}{4}x^4\right]_1^{e^2} - \int_1^{e^2} \frac{1}{x} \times \frac{x^4}{4}\,dx \quad ① \; ①$$

$\frac{1}{x} \times \frac{x^4}{4} = \frac{x^3}{4}$

$$= \left[\frac{x^4}{4}\ln x - \frac{x^4}{16}\right]_1^{e^2} \quad ①$$

$$= \left(\frac{e^{2^4}}{4}\ln(e^2) - \frac{e^{2^4}}{16}\right) - \left(\frac{1^4}{4}\ln 1 - \frac{1^4}{16}\right) \quad ①$$

$\ln e^2 = 2$

$\ln 1 = 0$

$$= \left(\frac{2e^8}{4} - \frac{e^8}{16}\right) - \left(-\frac{1}{16}\right)$$

$$= \frac{7}{16}e^8 + \frac{1}{16} \quad ①$$